2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 1–2 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

1. In square $ABCD$, point M is the midpoint of side $\overline{AB}$. Segments $\overline{MC}$ and $\overline{BD}$ meet at point P. Given that $CP = 1$, compute MP.

2. Given positive integers a, b, and c such that $a! \cdot b! \cdot c! = 12!$, compute all possible values of $a + b + c$.

ANSWER TO PROBLEM 1

ANSWER TO PROBLEM 2

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 3–4 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

3. The vertices of triangle $\mathcal{T}$ lie on the parabola $y = x^2$. Given that the slopes of the sides of $\mathcal{T}$ are -1, 2, and 7, compute the area of $\mathcal{T}$.

4. Compute the sum of all solutions to the equation $\cos(\cos(x)) = \cos(\sin(x))$ that lie in the interval $[0, 4\pi)$.

ANSWER TO PROBLEM 3

ANSWER TO PROBLEM 4

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 5–6 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

5. Consider the set of points in three-dimensional space that are within 2 units of the line segment $\overline{AB}$. Given that this set has volume $\frac{116\pi}{3}$, compute the length of segment $\overline{AB}$.

6. A five-digit number is selected at random. Given that its digits are all even, compute the probability that it is divisible by 3.

ANSWER TO PROBLEM 5

ANSWER TO PROBLEM 6

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 7–8 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

7. Compute the greatest prime factor of $(24!)^3 - (23!)^3$.

8. A non-degenerate convex quadrilateral has sides of lengths 2, 2, 3, and 4 in that order. Given that one of its diagonals has length 4, compute the length of the other diagonal.

ANSWER TO PROBLEM 7

ANSWER TO PROBLEM 8

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 9–10 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

9. Compute a, given that

$$\log_3(a) + \log_{3^3}(a) + \log_{3^9}(a) + \log_{3^{27}}(a) + \cdots = \sum_{n=0}^{\infty} \log_{3^{3^n}}(a) = 6.$$

10. An equiangular hexagon has sides of lengths 1, 2, 3, 4, 5, and 6 in some order. Compute the longest diagonal of any such hexagon.

ANSWER TO PROBLEM 9

ANSWER TO PROBLEM 10

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 11–12 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

11. Let $\mathcal{H}$ be a regular hexagon in the plane. Compute the number of equilateral triangles in the plane of $\mathcal{H}$ that share exactly two vertices with $\mathcal{H}$.

12. All angles in this problem are measured in degrees. Compute the least positive angle A such that

$$\sum_{n=1}^{2024} \cos(A + 40^n) = 1.$$

ANSWER TO PROBLEM 11

ANSWER TO PROBLEM 12

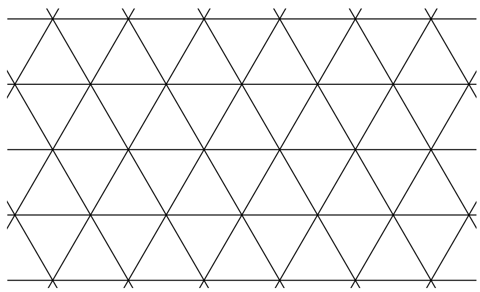
2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 13–14 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

13. The positive integers from 1 to 25 inclusive are placed into five disjoint sets, A, B, C, D, and E, each containing five integers. Let the medians of these sets be a, b, c, d, and e respectively. Let m be the median of $\{a, b, c, d, e\}$. Compute the least possible value for m.

14. A *regular triangular lattice* is like graph paper made from equilateral triangles:



Triangular lattice points are the intersections of the lines in such a lattice. Assuming the sides of the smallest triangles in the lattice have length 1, compute the radius of the smallest circle that is centered at a triangular lattice point that passes through 12 triangular lattice points.

ANSWER TO PROBLEM 13

ANSWER TO PROBLEM 14

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 15–16 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

15. Seven students are on the student council. The council has three subcommittees, each with three members; no two of the subcommittees consist of the same three students. Students who are on the same subcommittee shake hands before the council meetings starts. After the council meeting, every pair of students who haven't already shaken hands before the meeting do so now. Compute the ordered pair (m, M), where m is the minimum possible number of handshakes that occur after the meeting, and M is the maximum possible number of such handshakes.

16. Consider the function $f(x)$, defined for all nonzero real numbers, that satisfies the equation $2f(x) + 3f(1/x) = 7x$. Compute $f(7)$.

ANSWER TO PROBLEM 15

ANSWER TO PROBLEM 16

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 17–18 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

17. In $\triangle ACT$, $CA/CT = 5/8$, and the measure of angle C is $120°$. Point N is chosen on side $\overline{AT}$ so that $\overrightarrow{CN}$ is the bisector of $\angle C$. Compute CN/CA.

18. Complete the cross-number puzzle below, where each Across answer is a 4-digit number and each Down answer is a 3-digit number. No answer begins with the digit 0. **Your answer must be written in the grid at the bottom of the page; the grid to the right is only for scratch work!**

Across
1. A perfect cube
5. When read left to right, the digits form an increasing arithmetic progression
6. Exactly three of its digits are the same

Down
1. The square of the greatest prime factor of 2024
2. A perfect square
3. When read top to bottom, the digits form an increasing arithmetic progression
4. A perfect square

1	2	3	4
5			
6			

ANSWER TO PROBLEM 17

ANSWER TO PROBLEM 18

1	2	3	4
5			
6			

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 19–20 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

19. Compute the number of ordered triples of integers (a, b, c) with $a \le b \le c$ and $a+b+c \le 15$ such that both (a, b, c) and (a^2, b^2, c^2) are the side lengths of a non-degenerate triangle.

20. A *base-b palindrome* is a positive integer whose base-b representation reads the same forwards as backwards. For example, 121 is a base-ten palindrome and 89, which is 131 when written in base eight, is a base-eight palindrome. Compute the least positive integer greater than 6 that is both a base-five and base-six palindrome. (Report your answer in base ten.)

ANSWER TO PROBLEM 19

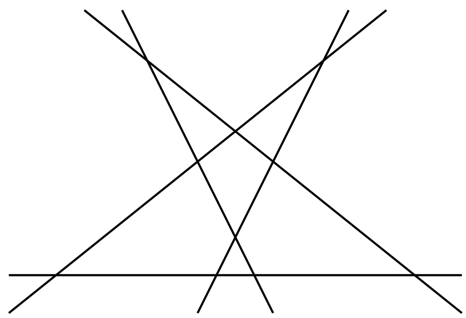
ANSWER TO PROBLEM 20

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 21–22 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

21. In the figure below there are five lines, no two of which are parallel and no three of which meet at a common point. If you count carefully you will find 10 different triangles in the figure. Compute n so that, if n lines are drawn in the plane with no two parallel and no three meeting at a common point, the figure will contain 2024 different triangles.



22. Point P lies inside equilateral $\triangle ABC$. Let X, Y, and Z be the feet of the perpendiculars from P to sides $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$ respectively. Given that $PX = 1$, $PY = 2$, and $PZ = 3$, compute $[APY] + [BPZ] + [CPX]$, where $[QRS]$ represents the area of $\triangle QRS$.

ANSWER TO PROBLEM 21

ANSWER TO PROBLEM 22

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 23–24 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

23. Everett is playing a game that uses a fair four-sided die with sides numbered 1–4, and a fair six-sided die with sides numbered 1–6. Everett starts with a score of zero by rolling the four-sided die. After any roll, if the number shown is prime, he adds that number to the score and continues the game by rolling the same die. If the number shown is composite, he subtracts that number from the score and continues the game by rolling the other die. If the number shown is 1, the game ends immediately. Compute the expected value of the score of Everett's game.

24. Compute the sum of the positive integers that are *not* in the domain of the real-valued function $\log_2(\log_2(\log_2(\log_2(\log_2(x)))))$.

ANSWER TO PROBLEM 23

ANSWER TO PROBLEM 24

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Part I Answers

1.	1/2 (or equivalent)	2.	14, 16 (must have both)
3.	60	4.	16π
5.	7	6.	417/1250
7.	601	8.	7/2 (or equivalent)
9.	81	10.	$\sqrt{67}$
11.	24	12.	260
13.	9	14.	$\sqrt{7}$

2024 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Part II Answers

15. $(12, 15)$

16. -19

17. $8/13$

18.

5	8	3	2
2	4	6	8
9	1	9	9

19. 22

20. 67

21. 24

22. $6\sqrt{3}$

23. $3/4$ (or equivalent)

24. 136

Tryout Results 2024

Item Analysis

($n = xxx$ for Q1–Q24)

Q	**# right**	**%**
1		%
2		%
3		%
4		%
5		%
6		%
7		%
8		%
9		%
10		%
11		%
12		%

Q	**# right**	**%**
13		%
14		%
15		%
16		%
17		%
18		%
19		%
20		%
21		%
22		%
23		%
24		%

Distribution of Top Scores

Score	$\boldsymbol{n}$
24	
23	
22	
21	
20	
19	
18	
17	
16	
15	
14	