Chicago Area All-Star Math Team

2024 Tryout Solutions

Names in parentheses at the end of each problem indicate the problem's author. CAASMT thanks all the problem authors for their contributions to this year's tryout!

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1. Triangles $\triangle PMB$ and $\triangle PCD$ are similar. Point M is given to be the midpoint of $\overline{AB}$, so $MB/CD = 1/2$. By similar triangles, MP/CP also equals $1/2$, and since $CP = 1$ that leaves $MP = \boxed{1/2}$. (Holden Mui)

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2. Assume without loss of generality that $a \le b \le c$. Since $12!$ has a factor of 11, one of $a!$, $b!$, or $c!$ must have a factor of 11. Since c is the largest of the three numbers, $c = 11$ or $c = 12$. The solution when $c = 12$ is obviously $a = 1$, $b = 1$, $c = 12$, whence $a + b + c = 14$. On the other hand, if $c = 11$ then $a! \cdot b! = 12$. This is possible if $a = 2$ and $b = 3$, making the total $a+b+c = 16$. These are the only possibilities: $\boxed{\text{14 and 16}}$. (Micah Fogel, inspired by ARML 2019 problem I9)

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3. Let the vertices of the triangle have coordinates (a, a^2), (b, b^2), and (c, c^2), with $a < b < c$. The slopes between these sets of points are $\dfrac{b^2 - a^2}{b - a} = b + a$, $\dfrac{c^2 - b^2}{c - b} = c + b$, and $\dfrac{c^2 - a^2}{c - a} = c + a$. Since $a < b < c$, these slopes are ordered $a + b < a + c < b + c$. So $a + b = -1$, $a + c = 2$, and $b + c = 7$. These three equations can easily be solved to find $a = -3$, $b = 2$, and $c = 5$.

Now the area of triangle $\mathcal{T}$ could be computed by a number of techniques. Heron's formula (extremely messy!) and the difference of the areas of the trapezoids with vertical bases and sides along the x-axis and sides of the triangle are two. The shoelace algorithm provides another. Using the trapezoidal method gives the area of the trapezoid with vertices $(-3, 9)$, $(-3, 0)$, $(5, 0)$, and $(5, 25)$ is $\frac{1}{2}(25 + 9) \cdot 8 = 136$, the trapezoid between $x = -3$ and $x = 2$ has area $\frac{1}{2}(9 + 4) \cdot 5 = \frac{65}{2}$, and the trapezoid between $x = 2$ and $x = 5$ has area $\frac{1}{2}(4 + 25) \cdot 3 = \frac{87}{2}$. The difference is $136 - \frac{65}{2} - \frac{87}{2} = \boxed{60}$. (Variation of a problem by Holden Mui)

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4. If $\cos(a) = \cos(b)$ then a and b either sum to or differ by an integer multiple of 2π. Let $a = \cos(x)$ and $b = \sin(x)$. Since $-1 \le a, b \le 1$ both their sum and difference lie between -2 and 2. Thus for $\cos(\cos(x)) = \cos(\sin(x))$ it must be the case that either $\cos(x) = \sin(x)$ or $\cos(x) = -\sin(x)$. More simply put, the only solutions to the equation are those x for which $\tan(x) = \pm 1$. In the interval $[0, 4\pi)$ these are $\pi/4$, $3\pi/4$, $5\pi/4$, $\ldots 15\pi/4$. These sum to $\boxed{16\pi}$. (Michael Caines)

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5. The set is a cylinder with radius 2 centered along the line segment, together with two hemispherical caps attached to its ends, also of radius 2. These two caps together make a sphere of radius 2, whose volume is $\frac{4}{3}\pi 2^3 = \frac{32}{3}\pi$. That means that the cylinder's volume is $\frac{84}{3}\pi = 28\pi$. Since the volume of a cylinder is $\pi r^2 h$ and $r = 2$ the height of the cylinder—the length of the line segment—must be $\boxed{7}$. (Michael Caines)

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6. There are four possible leading digits (2, 4, 6, 8) and five possibilities for each other digit, combining to yield $4 \cdot 5^4 = 2500$ five-digit numbers all of whose digits are even. Naively, 1/3 of these should be divisible by three. That is, 2500/3 of these should be, but that is not a whole number. Should it be rounded up or down? It is *not* simply the case that every third number among these numbers is divisible by three. For example, 20006, 20008, 20020 are three such numbers in a row that are not divisible by three, while 20028 and 20040 are two consecutive such numbers that both *are* divisible by three. Counting must proceed more carefully.

Consider any position within the number. There are exactly as many numbers where the digit in that position is a 2 as there are where that digit is a 4 or a 6. Exactly one of those three numbers will be divisible by 3. So among all the numbers where at least one of the digits is not 8 or 0, exactly one third of them are divisible by 3. Since the first digit cannot be 0, there are only 16 numbers where all the digits are 8 or 0. So among the $2500 - 16 = 2484$ numbers where not all the digits are 8 or 0, 828 of them are divisible by 3.

If all the digits are either 8 or 0, for the number to be divisible by 3 there must be three 8's and two 0's. Those two 0's can be in any of four positions, so there are $\binom{4}{2} = 6$ numbers consisting of only 8's and 0's that are divisible by 3. Added to the 828 numbers we have where not all the digits are 8 or 0, there are 834 5-digit numbers, all of whose digits are even, that are divisible by 3. Since there are 2500 candidate numbers, the probability of choosing one of the multiples of 3 at random is $834/2500 = \boxed{417/1250}$. (Holden Mui)

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7. The expression can be rewritten as $(24 \cdot 23!)^3 - (23!)^3$ from which $(23!)^2$ can be factored from both terms:

$$(24!)^3 - (23!)^3 = (23!)^3(24^3 - 1).$$

The largest prime factor of 23! is 23. What about the other term? Using the difference-of-cubes formula $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ gives $24^3 - 1 = (24-1)(24^2+24+1) = 23 \cdot 601$. In fact, $\boxed{601}$ is a prime. This can be seen because it is not divisible by 2, 3, 5, 7, 11, 13, 17, 19, or 23, which are the only primes less than its square root. (Micah Fogel, based on an old tryout problem)

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8. By the triangle inequality, the given diagonal must go between the vertex where the two sides of length 2 meet and the vertex where the sides of length 3 and 4 meet, otherwise the quadrilateral would be degenerate. Now, this problem would be a lot easier if you knew that the quadrilateral was cyclic, for then Ptolemy's theorem the product of the diagonals equals the sum of the products of the pairs of opposite sides. In this case, $4x = 2 \cdot 3 + 2 \cdot 4$ where x is the unknown diagonal. So if the quadrilateral were cyclic the unknown diagonal would have length $\boxed{7/2}$.

In fact, if you were running out of time and had to make a guess, it is very reasonable to guess that the quadrilateral is cyclic and solve the problem as above. It wouldn't be any more wrong than to leave the question blank after all! But fortunately the quadrilateral *is* cyclic. This can be easily checked using the law of cosines. The diagonal splits the quadrilateral into a 2-3-4 triangle and a 2-4-4 triangle. Using the law of cosines on the angle opposite the common diagonal in the 2-3-4 triangle gives

$$4^2 = 2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cos(\theta_1)$$

so that $\cos(\theta_1) = -1/4$. Similarly the angle opposite the common diagonal in the 2-4-4 triangle leads to

$$4^2 = 2^2 + 4^2 - 2 \cdot 2 \cdot 4 \cos(\theta_2)$$

so that $\cos(\theta_2) = 1/4$. Since the two cosines are opposites, the angles must be supplemntary. A quadrilateral with supplementary opposite angles is cyclic.

Incidentally, there is an extension of Brahmagupta's formula for the area of a quadrilateral given the four sides, a, b, c, and d, and the two diagonals, p and q. Letting $s = (a+b+c+d)/2$ be the semiperimeter, *Coolidge's formula* states that the area of the quadrilateral is given by

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d) - \frac{1}{4}(ac+bd+pq)(ac+bd-pq)}.$$

The formula reduces to Brahmagupta's formula in case of a cyclic quadrilateral, because Ptolemy's theorem causes the final parenthesized expression to be zero. A mathlete who knows Coolidge's formula could solve this problem by using Heron's formula to find the area of each of the two triangles, adding their areas, and solving for the remaining diagonal using Coolidge.

An alternate approach makes extensive use of the law of cosines. The given diagonal splits the quadrilateral into a 2-4-4 triangle and a 2-3-4 triangle. In the 2-4-4 triangle, the cosine of the angle (call it α) between the side of length 2 and the diagonal of length 4 can be found from the law of cosines: $4^2 = 4^2 + 2^2 - 2 \cdot 2 \cdot 4\cos(\alpha)$, so $\cos(\alpha) = 1/4$. For the 2-3-4 triangle, the angle (call it β) between the side of length 2 and the diagonal can be found from $3^2 = 4^2 + 2^2 - 2 \cdot 2 \cdot 4\cos(\beta)$, so $\cos(\beta) = 11/16$. The sines of these angles are then $\sin(\alpha) = \sqrt{15}/4$ and $\sin(\beta) = 3\sqrt{}/16$. Use the angle addition formula to find $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) = 11/64 - 45/64 = -17/32$. Then one more use of the law of cosines computes the other diagonal (call its length d) from $d^2 = 2^2 + 2^2 - 2 \cdot 2 \cdot 2 \cdot (-\frac{17}{32}) = \frac{49}{4}$, and taking the square root yields the result.

A third approach to the problem would be to set the quadrilateral in a Cartesian coordinate system. The vertex where the two sides of length 2 meet could be set at the origin $(0,0)$, while the other end of the given diagonal would placed at $(4,0)$. The other two vertices' locations could be found by setting up systems of equations. The point where the sides of lengths 2 and 3 meet would be at one intersection (say, the one with positive y-coordinate) of the circles $x^2+y^2 = 2^2$ and $(x-4)^2+y^2 = 3^2$, which is $(11/8, 3\sqrt{15}/8)$. The point where the sides of lengths 2 and 4 meet would be the intersection (with negative y-coordinate) of the circles $x^2+y^2 = 2^2$ and $(x-4)^2+y^2 = 4^2$, at $(1/2, -\sqrt{15}/2)$. The distance formula then finishes the problem. (Holden Mui)

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9. Using the base-change formula for logarithms, $\log_{3^{3^n}}(a) = \dfrac{\log_3(a)}{\log_3(3^{3^n})} = \dfrac{\log_3(a)}{3^n}$. Thus the sum in question becomes the infinite geometric series $\log_3(a)\left(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots\right)$. This sums to $\frac{3}{2}\log_3(a)$. So $\frac{3}{2}\log_3(a) = 6$ or $\log_3(a) = 4$. Thus $a = 3^4 = \boxed{81}$. (Micah Fogel)

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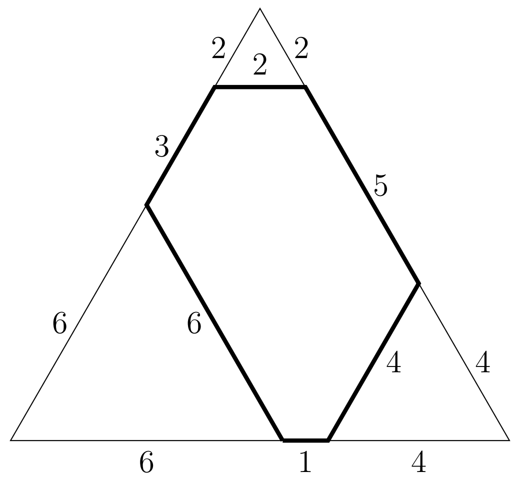
10. First, a brute force solution will be shown, followed by a much more elegant approach. For the brute force solutions, place the hexagon in the plane so that the side of length 1 has its end points at the origin and at $(1,0)$, and so that the interior of the hexagon lies in the upper half-plane. Starting at the origin, moving around the hexagon in a counterclockwise direction, let the sides have lengths 1, a, b, c, d, and e in that order. The side of length c is parallel to the x-axis, lying along the line $y = (a+b)\sqrt{3}/2$. Traveling clockwise from the origin, this side can be seen to lie along the line $y = (d+e)\sqrt{3}/2$. Thus $a+b = d+e$. In similar fashion, the sum of the lengths of any two consecutive sides of the hexagon must equal the sum of the lengths of the two opposite sides. Once this is discovered, it can quickly be determined that the only possible orders of the sides are 145236 and 153426 (and their rotations and reflections). Determine the lenghts of the diagonals of these two hexagons.

In the hexagon whose sides are in the order 145236, the vertices of the hexagon (if positioned as above) are at $(0,0)$, $(1,0)$, $(3, 2\sqrt{3})$, $(1/2, 9\sqrt{3}/2)$, $(-3/2, 9\sqrt{3}/2)$, and $(-3, 3\sqrt{3})$. The largest distance between any of these points is $\sqrt{67}$, between the second and fifth point. For the 153426 hexagon, the coordinates are $(0,0)$, $(1,0)$, $(7/2, 5\sqrt{3}/2)$, $(2, 4\sqrt{3})$, $(-2, 4\sqrt{3})$, and $(-3, 3\sqrt{3})$. The largest distance between a pair of these points is $\sqrt{57}$. So the longest diagonal such a hexagon can have is $\boxed{\sqrt{67}}$.

The more elegant approach is to extend three non-adjacent sides of the hexagon until they meet, creating an equilateral triangle. This is shown below for the hexagon with sides in the order 145236:



Drawing this triangle shows that the sum of any two adjacent sides of the hexagon must equal the sum of the two opposite sides, because both equal the length of the side of the equilateral triangle minus the length of one of the intervening sides of the hexagon. Then, once the two possible arrangements of lengths of the sides that create hexagons are determined, the law of cosines can be applied to quickly find the lengths of the diagonals of the hexagon. (Holden Mui)

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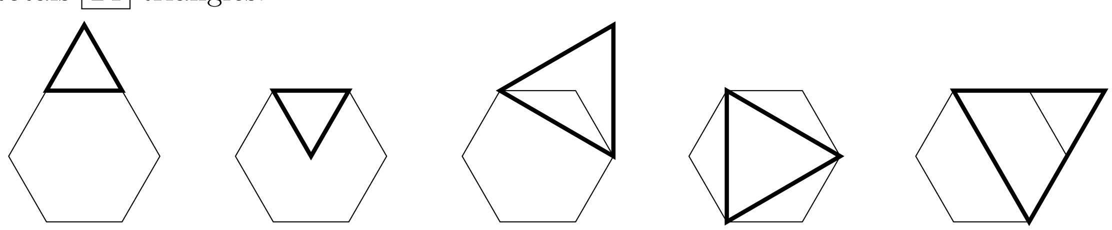
11. There are six such triangles that share an edge with $\mathcal{H}$ and are external to it. There are another six that share an edge with $\mathcal{H}$ and are internal to it. One example of each of these is shown in the first diagram below, and the others are rotations of these figures around the other edges of the hexagon. As shown in the third diagram, there are also six such triangles that share a pair of non-adjacent, non-opposite vertices that "point outward." However, as shown in the fourth diagram below there aren't any that "point inward," because then they would share a third vertex with H. Finally, there are six more such triangles what share a pair of opposite vertices, shown in the fifth diagram. Altogether, this totals $\boxed{24}$ triangles.



(Micah Fogel, based on a problem by Andy Niedermaier)

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12. Notice that $40^2 = 1600$ and an angle of $1600°$ is equivalent to an angle of $160°$.

Multiplying by 40 again gives an angle of 6400°, which is equivalent to an angle of 280°. Multiplying by 40 one more time returns to an angle of 40°. In other words, $A+40^n$ is the same angle as $A+40+120(n-1)$. Thus for any A, the angles in this problem repeatedly represent three points on the unit circle that form an equilateral triangle.

Taking cosines of these angles and adding is equivalent to adding the x-coordinates of the vertices of an equilateral triangle centered at the origin. Since the centroid of such a triangle is the origin, the x-coordinates of its vertices must average—and hence add—to zero. Thus, the terms of the series cancel in groups of three, leaving just the final two terms. These two terms simplify to $\cos(A+40)+\cos(A+160)$. Now the only two angles that are separated by 120° whose cosines add to 1 are the points at ±60°. So $A+40=-60$ or $A=-100$. Since the problem required the least positive A, add 360 to obtain $A=\boxed{260}$. (Micah Fogel, based on a problem by Mark Fritz)

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13. Without loss of generality, assume that $a<b<c<d<e$ so that $m=c$. In C, $m=c$ is larger than two of the other values in that set. Also, m is larger than at least three of the numbers is A (since $m>a$ and a is the median of five-element set A). Similarly, m is larger than at least three values in B. Thus, m is larger than at least eight of the numbers from 1–25. So $m\geq 9$. Below is one possible partition showing that $m=\boxed{9}$ can be obtained. The goal is to make the "at least" in the foregoing sentences into "exactly":

$$\begin{aligned} A &= \{1,2,3,10,11\} \\ B &= \{4,5,6,12,13\} \\ C &= \{7,8,9,14,15\} \\ D &= \{16,17,18,19,20\} \\ E &= \{21,22,23,24,25\}. \end{aligned}$$

(Micah Fogel, based on a problem by Andy Niedermaier)

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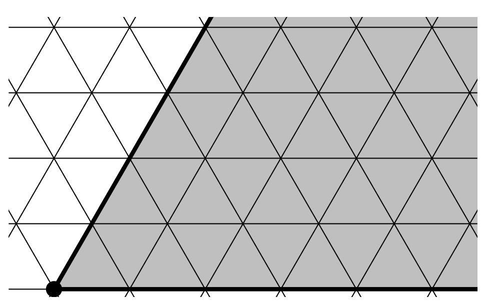
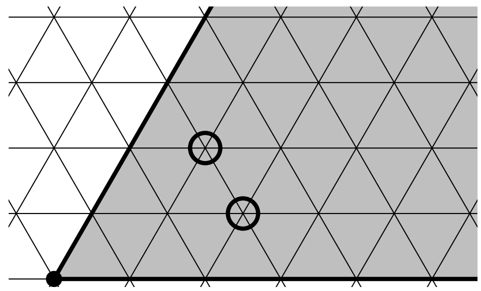
14. Consider a triangular "sextant" originating from the center of the circle:



By symmetry, if there are 12 points of the circle that are lattice points of the triangular grid, either there must be two such points within each sextant, or one point in each sextant and one point on each of the six lines dividing the plane into sextants. It should be clear that the two circled points in the diagram below are the two closest points to the center that satisfy one or the other of these requirements.



Using the geometry of equilateral triangles, it is straightforward to compute that these two points are $\boxed{\sqrt{7}}$ units from the center of the circle. (Holden Mui)

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15. Before the meeting, each different subcommittee causes three handshakes to occur, one for each pair of its members. However, if two students are both members of two different subcommittees then their before-meeting handshakes only remove one from the number that must occur after the council meeting. Thus, to find m, determine the way to get the least number of overlapping subcommittee members. If the students are (A, B, C, D, E, F, G) then the three subcommittees (A, B, C), (D, E, F), and (A, D, G) will use up nine handshakes before the council meeting starts. Since there are seven members of the council, there would be $\binom{7}{2} = 21$ handshakes in total, so $m = 21 - 9 = 12$.

On the other hand, to find M determine the maximum possible overlap of the subcommittees. One way to do this would be to have subcommittees (A, B, C), (A, B, D), and (B, C, D). This only uses up six of the handshakes, leaving $M = 15$ handshakes for after the meeting.

Combining these pieces of information, $(m, M) = \boxed{(12, 15)}$. (Mark Fritz)

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16. Substituting $x = 7$ gives $2f(7) + 3f(1/7) = 49$, while substituting $x = 1/7$ leads to $2f(1/7) + 3f(7) = 1$. Adding these two results gives $5(f(7) + f(1/7)) = 50$ or $f(7) + f(1/7) = 10$. Subtracting them results in $f(7) - f(1/7) = -48$. Adding these last two results cancels the undesired term, leaving $2f(7) = -38$ or $f(7) = \boxed{-19}$. (Mark Fritz)

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17. There are a number of ways of approaching this problem, such as using the law of cosines or Stewart's Theorem to explicitly compute the length CN. These are messy; here are two much more elegant approaches.

First, by the angle bisector theorem $AN/TN = 5/8$; equivalently $TN/AT = 8/13$. The area of $\triangle ACT$ is $\frac{1}{2}\cdot CA\cdot CT\cdot\sin(120°)$ while the area of triangle $\triangle NCT$ is $\frac{1}{2}\cdot CN\cdot CT\cdot\sin(60°)$. Now since $\sin(60°) = \sin(120°)$, dividing these expressions shows that CN/CA is equal to the ratio of the areas of triangles $\triangle NCT$ and $\triangle ACT$. This ratio can also be computed by observing that these triangles can be viewed as having bases $\overline{NT}$ and $\overline{AT}$ respectively, with the same altitude. So the ratio of the areas is the same as the ratio of the bases, $\boxed{8/13}$.

For an even more elegant approach, once $TN/AT = 8/13$ has been established, consider that $\overrightarrow{CA}$ is an exterior angle bisector for triangle $\triangle NCT$. Thus by the exterior angle bisector theorem, $CN/CA = TN/AT = \boxed{8/13}$. (Michael Caines)

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18. Starting with 1-Down, the prime factors of 2024 are 2, 11 and 23; the square of the largest is $23^2 = 529$. Now 1-Across is a 4-digit cube with first digit 5. Since $10^3 = 1000$ and $20^3 = 8000$ search between these two values. In fact, $16^3 = 2^{12} = 4096$ so it needs to be a little larger. Checking, $17^3 = 4913$ is too small, while $18^3 = 5832$, so 19^3 will be too large. So 1-Across is 5832.

Since 2-Down is a square with first digit 8, it must be $29^2 = 841$. Thus 5-Across is 2468, and 3-Down is 369. The only possibility for 6-Across is now 9199. This is confirmed because 4-down reads 289, which is 17^2. (Chris Jeuell)

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19. The triangle inequality says that triple (x, y, z) with $x \leq y \leq z$ is the side-lengths of a non-degenerate triangle is and only if $z - y < x$. So if one side has length 1 the other two sides must be equal, leading to seven such triples that add to at most 15: $(1, 1, 1)$, $(1, 2, 2)$, $(1, 3, 3)$,. . . ,$(1, 7, 7)$.

If the shortest side of the triangle is 2, the other two sides must still be equal. For in this case, $c^2 - b^2 = (c + b)(c - b)$ and since b and c are both at least 2, $b + c \geq 4$. So if $c - b$ is not zero, $c^2 - b^2 \geq 4 = 2^2$. So the squares cannot be made into a triangle. Thus, five more triples are obtained: $(2, 2, 2)$, $(2, 3, 3)$, $(2, 4, 4)$, $(2, 5, 5)$, and $(2, 6, 6)$.

For $a = 3$, again the cases of $b = c$ will work, but in addition the triple $(3, 3, 4)$ works, because $4^2 - 3^2 = 7 < 9^2$ also forms a triangle. So five more triples.

For $a = 4$ it can be quickly checked that $(4, 4, 4)$, $(4, 4, 5)$, $(4, 5, 5)$, and $(4, 5, 6)$ all work. Of course, $(5, 5, 5)$ is the only triple adding to at most 15 when the smallest number is 5.

In total, this amounts to $7 + 5 + 5 + 4 + 1 = \boxed{22}$ triples. (Micah Fogel)

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20. Since the number is greater than 6, it is greater than 11 written in base-five. The next few base-five palindromes are 12, 18, and 24 (22, 33, and 44 in base-five), none of which are base-6 palindromes. Number which are three-digit palindromes in base five have the form aba in base five, so are of the form $26a + 5b$ in base ten, with $0 \leq a, b \leq 4$. On the other hand, two-digit palindromes in base six have the form cc in base six, so are of the form $7c$ in base ten, with $0 \leq c \leq 6$. There are no multiples of 7 among the number of the form $26a + 5b$ less than $42 = 7 \cdot 6$, so there are no numbers which are three-digit base-5 palindromes which are two-digit base-6 palindromes.

Trying three-digit base-6 palindromes, these are of the form cdc in base six, so $37c+6d$ in base ten, with $0 \leq c, d \leq 6$. Checking possible values for c and d it will quickly be discovered that $37 \cdot 1 + 6 \cdot 5 = 26 \cdot 2 + 5 \cdot 3 = 67$, so that $\boxed{67}$ is a both a base-six palindrome (with base-6 representation 151) and a base-five palindrome (with base-5 representation 232). (Holden Mui)

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21. Since no two lines are parallel, each pair of lines meets at some point. Then, since no three lines meet at a common point, any three lines determine a triangle. Thus, if there are n lines, there will be $\binom{n}{3}$ triangles. Note that $2024 = 8 \cdot 11 \cdot 23 = 4 \cdot 22 \cdot 23$. A little tinkering converts this to $\dfrac{22 \cdot 23 \cdot 24}{6} = \binom{24}{3}$. Thus $n = \boxed{24}$. (Micah Fogel)

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22. The solution to this problem combines two nifty theorems about equilateral triangles. The first is that, *regardless* of the location of point P within $\triangle ABC$, the three triangles $\triangle APY$, $\triangle BPZ$, and $\triangle CPX$ have the same total area as the three triangles $\triangle APZ$, $\triangle CPY$, and $\triangle BPX$. A full proof will not be given here, but if you are curious try drawing segments $\overline{A'B'}$, $\overline{B'C'}$ and $\overline{A'C'}$ parallel to the sides of the original triangle and look at the various triangles and parallelograms into which the original triangle $\triangle ABC$ has now been partitioned. From this theorem, the total area requested is half the area of $\triangle ABC$.

This is where the second theorem comes it. *Viviani's Theorem* states that *regardless* of the location of point P within an equilateral triangle, the total of the lengths of the three perpendiculars $\overline{PX}$, $\overline{PY}$, and $\overline{PZ}$ is constant. This is easily proved because the sum of the areas $[PAB] + [PBC] + [PCA]$ must equal the area of the original triangle $\triangle ABC$ and the altitudes of these triangles are PZ, PX, and PY respectively while all three have the same base, $AB = BC = AC$. Since the sum of the three perpendicular from any point inside $\triangle ABC$ is 6, by moving this point arbitrarily close to the midpoint of one of the sides of $\triangle ABC$ it can be seen that the altitude of $\triangle ABC$ must be 6. An equilateral triangle who altitude is 6 has area $12\sqrt{3}$ so half that area is $\boxed{6\sqrt{3}}$. (Holden Mui)

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23. Let A be the expected value of the score if the game is started by rolling the four-sided die, and B be the expected value of the score when the game starts by rolling the six-sided die. Since Everett starts by rolling the four-sided die, the answer to this question will be A.

By linearity of expectations,

$$A = \frac{1}{4}(0) + \frac{1}{4}(2 + A) + \frac{1}{4}(3 + A) + \frac{1}{4}(-4 + B).$$

This simplifies to $\frac{1}{2}A = \frac{1}{4}(1+B)$. Similarly,

$$B = \frac{1}{6}(0) + \frac{1}{6}(2+B) + \frac{1}{6}(3+B) + \frac{1}{6}(-4+A) + \frac{1}{6}(5+B) + \frac{1}{6}(-6+A).$$

This can be simplified to $\frac{1}{2}B = \frac{1}{3}A$ or $B = \frac{2}{3}A$. Substituting this into the equation found above and solving for A yields $A = \boxed{3/4}$. (Michael Caines)

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24. As long as $\log_2(\log_2(\log_2(\log_2(x)))) > 0$ the outer-most logarithm is defined. But this requires $\log_2(\log_2(\log_2(x))) > 1$, which in turn makes $\log_2(\log_2(x)) > 2$. Continuing, $\log_2(x) > 4$, so $x > 16$. Thus, the positive integers 1–16 are not in the domain, and their sum is $16 \cdot 17/2 = \boxed{136}$. (Michael Caines)