2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 1-2 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

1. Compute $\sqrt{(17)(18)(19)(20)+1}$.

2. Three standard, fair six-sided dice are rolled. Given that the sum of the values rolled is 11, compute the probability that *none* of the numbers showing is prime.

ANSWER TO PROBLEM 1

ANSWER TO PROBLEM 2

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 3-4 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

3. A right equilateral triangular prism has triangular faces of side length 10 cm, and is 20 cm long. It is placed on one of the rectangular faces and filled to half its volume with water. Compute the area of the water's (top) surface in cm^2.

4. Compute the sum of the (complex) roots of the polynomial

$$(x-1)^{2020} + (x+2)^{2020} + (x-3)^{2020} + (x+4)^{2020} + \cdots + (x-2019)^{2020} + (x+2020)^{2020}.$$

ANSWER TO PROBLEM 3

ANSWER TO PROBLEM 4

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 5-6 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

5. In $\triangle LVS$, point H lies on $\overline{VS}$ so that $\overline{LH}$ bisects $\angle VLS$ and $HS = 5$. Given that $LV = 14$ and VH and LS are both integers, compute the least possible perimeter of $\triangle LVS$.

6. Seven tiles with the letters S, T, R, E, E, T, S are arranged on the vertices of a regular heptagon. Compute the probability that beginning with the letter R and proceeding either clockwise or counterclockwise without skipping vertices, the letters spell out the word *RETESTS*.

ANSWER TO PROBLEM 5

ANSWER TO PROBLEM 6

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 7-8 **Name** ______________________________

Time Limit: 10 minutes **School** ______________________________

7. For a positive real number k, let R_k be the region above the x-axis such that $|x| + |y| \geq k$ and $|2x| + |y| \leq 2k$. Given that the area of R_k is 34, compute k.

8. Given that the system of equations below has no solutions, compute the value of $a + b$.

$$\begin{cases} 3x + 8y + 13z &= 14 \\ 7x + 3y - z &= 12 \\ ax + 17y + bz &= 57 \end{cases}$$

ANSWER TO PROBLEM 7 **ANSWER TO PROBLEM 8**

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 9-10 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

9. Two rectangular pools Y and P are surrounded by rectangular walkways of width w feet. The sides of Y are each six feet longer than the corresponding sides of P, and the area of the walkway surrounding Y is 150 square feet more than the area of the walkway surrounding P. Compute w.

10. Set A consists of m consecutive integers whose sum is $2m$, and set B consists of $2m$ consecutive integers whose sum is m. The absolute value of the difference between the greatest element of A and the greatest element of B is 17. Compute m.

ANSWER TO PROBLEM 9

ANSWER TO PROBLEM 10

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 11-12 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

11. A bag contains eight tiles numbered $1, 2, 3, \ldots, 8$. Xander, Aubrey, Eamon, and Neala each take two tiles from the bag without looking (and without replacing the tiles), and add the numbers on their tiles. Compute the probability that all four sums are odd.

12. Let P be the point $(22, 4)$. The negatively-sloped tangent from point P to the circle centered at $(5, 4)$ with radius 8 intersects the circle at point $Q(x, y)$. Compute x.

ANSWER TO PROBLEM 11

ANSWER TO PROBLEM 12

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 13-14 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

13. Ari, Jonah, and Helen inherit their grandfather's flock of n emus. According to the will, Ari is to receive $1/2$ of the emus, Jonah is to receive $1/3$ of the emus, and Helen is to receive $1/h$ of the emus, where h is a positive integer. Unfortunately, n is not divisible by 2, 3, or h, and individual emus aren't amenable to division, so the children are stuck until a neighbor gives them an emu. The increased flock is much easier to divide: Ari gets exactly $1/2$ of the emus, Jonah gets exactly $1/3$ of the emus, and Helen gets exactly $1/h$ of the emus. Even better, there is exactly one emu left over, which they give back to the neighbor. Compute the greatest possible value of n.

14. Let n be the greatest positive integer that, when expressed in base 10, has the following property: each consecutive pair of digits, reading from left to right, forms a positive perfect square. Compute n.

ANSWER TO PROBLEM 13

ANSWER TO PROBLEM 14

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 15-16 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

15. Compute the number of lattice points on the graph of $y = (x - 187)^{187 - x^2}$.

16. Given that a and b are real numbers satisfying $\log_8 a^2 + \log_4 b^3 = 6$ and $\log_4 a^3 + \log_8 b^2 = 7$, compute ab.

ANSWER TO PROBLEM 15

ANSWER TO PROBLEM 16

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 17-18 **Name** ____________________________

Time Limit: 12 minutes **School** ____________________________

17. The integer n is the least positive integer divisible by 165 whose base-10 representation consists only of 2's and 0's. Compute n.

18. Complete the cross-number puzzle below, where each Across answer is a four-digit number and each Down answer is a three-digit number. No answer begins with the digit 0.
Your answer must be written in the space at the bottom of this page, not the grid to the right!

<u>Across</u>
1. A product of three consecutive integers
5. A multiple of 2020
6. A perfect cube

<u>Down</u>
1. A Fibonacci number
2. Eleven less than a factorial
3. A palindrome
4. Each digit is a multiple of 3

1	2	3	4
5			
6			

ANSWER TO PROBLEM 17

ANSWER TO PROBLEM 18

1	2	3	4
5			
6			

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 19-20 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

19. Five people take turns rolling a fair six-sided die numbered 1 through 6 once each. Compute the probability that each person's roll is no lower than the previous person's roll.

20. Given that $2\tan^{-1}(x) + \tan^{-1}(2x) = \dfrac{\pi}{2}$, compute x^2.

ANSWER TO PROBLEM 19

ANSWER TO PROBLEM 20

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 21-22 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

21. Every day, Sheila and Mary go to the same coffee shop for a cup of coffee after work. Each person arrives at a random time between 4 pm and 5 pm and stays for exactly m minutes. Given that the probability that the two people meet is exactly $1/2$, compute m.

22. Point L lies inside triangle ARM so that $AL = 8$, $RL = 5$, and $\mathrm{m}\angle ALM = \mathrm{m}\angle ALR = \mathrm{m}\angle MLR = 120^\circ$. Given that $\angle ARM$ is a right angle, compute LM.

ANSWER TO PROBLEM 21

ANSWER TO PROBLEM 22

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 23-24 **Name** ____________________________

Time Limit: 10 minutes **School** ____________________________

23. For each integer n, let $c(n)$ be the least positive integer such that $n \cdot c(n)$ is a perfect cube. Compute the least positive integer n such that $c(n) = 2020$.

24. Given that A, R, M, L are positive integers (not necessarily distinct) such that $A^2 + R^2 = M^2 - L^2 = 20$, compute the greatest possible value for the sum $A + R + M + L$.

ANSWER TO PROBLEM 23

ANSWER TO PROBLEM 24

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Part I Answers

1. 341
2. $\frac{2}{9}$
3. $100\sqrt{2}$
4. -1010
5. 36
6. $\frac{1}{45}$
7. $\sqrt{34}$
8. 34
9. $\frac{25}{4}$ or $6\frac{1}{4}$ or 6.25
10. 37
11. $\frac{8}{35}$
12. $8\frac{13}{17}$ or $\frac{149}{17}$
13. 41
14. 81649

2020 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Part II Answers

15. 29

16. 64

17. 2222220

18.

1	7	1	6
4	0	4	0
4	9	1	3

19. $\frac{7}{216}$

20. $\frac{1}{5}$ or 0.2

21. $60 - 30\sqrt{2}$

22. 30

23. 510050

24. 16

Tryout Results 2020

Item Analysis

($n = 186$ for Q1-Q24)

Q	# right	%
1	119	64%
2	123	66%
3	125	67%
4	83	45%
5	89	48%
6	72	39%
7	125	67%
8	166	89%
9	154	83%
10	40	22%
11	113	61%
12	40	22%

Q	# right	%
13	125	67%
14	135	73%
15	80	43%
16	28	15%
17	110	59%
18	59	32%
19	47	25%
20	22	12%
21	49	26%
22	128	69%
23	121	65%
24	98	53%

Distribution of Top Scores

Score	n
24	2
23	5
22	4
21	7
20	5
19	6
18	9
17	11
16	12
15	11
14	16