2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 1-2 **Name**________________________________

Time Limit 8 Minutes **School**_______________________________

1. In one week, a salesman sells p cars for q thousands of dollars each, earning a commission of $100 plus q% of the selling price each time he sells a car, where p and q are positive integers. If the salesman earns $4600 for the week, compute the ordered pair (p, q).

2. If the slope of the line between $(-2, x^2 + 2x)$ and $(x, 4x + 8)$ is $-\frac{3}{4}$, compute the value of x.

ANSWER TO PROBLEM 1

ANSWER TO PROBLEM 2

2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 3-4 **Name**________________________________

Time Limit: 9 Minutes **School**________________________________

3. Compute the value (in simplest form) of $\left(36+10\sqrt{11}\right)^{3/2} - \left(36-10\sqrt{11}\right)^{3/2}$.

4. If $x > 1$, compute the minimum positive value of $\log_2 x^2 + \log_x 4$.

ANSWER TO PROBLEM 3

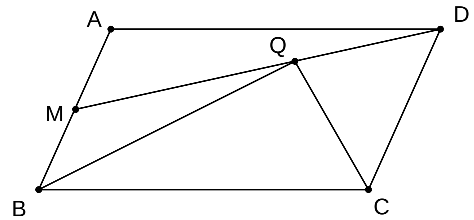
ANSWER TO PROBLEM 4

2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 5-6 **Name**________________________________

Time Limit: 10 Minutes **School**________________________________

5. In parallelogram *ABCD*, *CD* = 12, *AM* = 6, and Δ *QMB* and Δ *DQC* each have area 60. Compute the area of Δ *QBC*.



6. Compute the smallest positive value of x (in radians) such that $\dfrac{\sin x - \sin 7x}{\cos 7x - \cos x} = \tan 6x$.

ANSWER TO PROBLEM 5 **ANSWER TO PROBLEM 6**

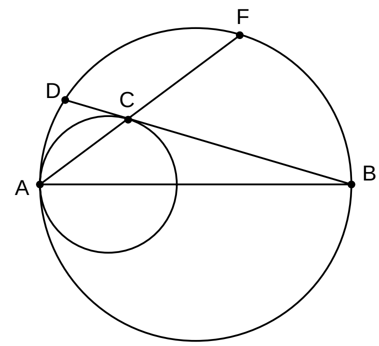
2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 7-8 **Name**________________________________

Time Limit: 10 Minutes **School**________________________________

7. If $f\left(\dfrac{2x-3}{x-2}\right) = 5x-2$, compute $f^{-1}(27)$.

8. Two circles are internally tangent at point A; diameter $\overline{AB}$ of the larger circle passes through the center of the smaller circle. Chord $\overline{BD}$ of the larger circle is tangent to the smaller circle at C; $\overleftrightarrow{AC}$ intersects the larger circle at F. If the measure of arc $\overset{\frown}{FB} = 82°$, compute the measure of arc $\overset{\frown}{AD}$.



ANSWER TO PROBLEM 7

ANSWER TO PROBLEM 8

2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 9-10 **Name**________________________________

Time Limit: 10 Minutes **School**________________________________

9. Two teams of three swimmers are in a pool with seven lanes. No two swimmers from the same team are allowed to be in adjacent lanes. Compute the number of possible arrangements if the swimmers themselves are distinguishable.

10. The first term of an infinite geometric series is 21. The second term and the sum of the series are both positive integers. Compute all possible values of the second term.

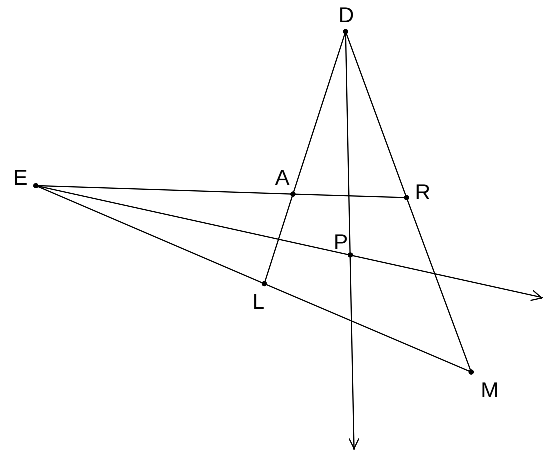
ANSWER TO PROBLEM 9 **ANSWER TO PROBLEM 10**

2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 11-12 **Name**________________________________

Time Limit: 12 Minutes **School**_______________________________

11. In convex quadrilateral *ARML*, sides $\overline{AL}$ and $\overline{RM}$ are extended to point *D*, and sides $\overline{AR}$ and $\overline{ML}$ are extended to point *E*. Both points *D* and *E* lie outside the quadrilateral. Bisectors of angles *E* and *D* are drawn, intersecting at point *P*. If m$\angle$ *DPE* = 72° and m$\angle$ *ARM* = 114°, compute m$\angle$ *ALM*.



12. If sin(sin *x* + cos *x*) = cos(cos *x* – sin *x*), compute the largest possible value of sin *x*.

ANSWER TO PROBLEM 11

ANSWER TO PROBLEM 12

2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 13-14 **Name**________________________________

Time Limit: 9 Minutes **School**________________________________

13. Some lines are drawn in the plane, no two parallel and no three concurrent. Another k lines are drawn in the same plane, parallel to each other but not to any of the original lines and again with no three lines concurrent. If the total number of lines altogether is 8 and the lines form a total of 40 triangles, compute k. Note that triangles whose areas overlap, but which have some areas not in common, are considered distinct.

14. Compute the number of positive integers b such that the equation $12x + by = 42$ has solutions (x, y) where both x and y are positive integers.

ANSWER TO PROBLEM 13

ANSWER TO PROBLEM 14

2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 15-16 **Name**________________________________

Time Limit: 11 Minutes **School**________________________________

15. If n is a positive integer such that n^2 has 2009 distinct positive integer divisors, compute the smallest number of distinct positive integer divisors that n could have.

16. Five distinct numbers from the set {1, 2, 3, … , 17} are chosen at random. Compute the probability that their sum is divisible by 3?

ANSWER TO PROBLEM 15

ANSWER TO PROBLEM 16

2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 17-18 **Name_________________________________**

Time Limit: 12 Minutes **School_________________________________**

17. If $4x^3 - 3x^2 - 12x + 23 = a(x+2)^3 + b(x+2)^2 + c(x+2) + d$, compute $a + b + c + d$.

18. Complete the cross-number puzzle, where each Across answer is a 4-digit number and each Down answer is a 3-digit number. No answer begins with the digit 0. **NOTE: Your answer must be written in the spaces at the bottom of this page, not the grid to the right of the clues.**

Across
1. All but one of its digits are the same.
5. All of its digits are different.
6. Its digits are strictly decreasing

Down
1. A power of 2.
2. A factor of 2009.
3. Twice a perfect square.
4. A perfect square.

1	2	3	4
5			
6			

ANSWER TO PROBLEM 17

ANSWER TO PROBLEM 18

1	2	3	4
5			
6			

2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 19-20 **Name**________________________________

Time Limit: 11 Minutes **School**________________________________

19. Point A is on the bisector of $\angle BOC$, whose measure is 60°; $OA = 12$ units. Compute the minimum perimeter of ΔABC.

20. A complex number $z = a + bi$, $b > 0$, has the property that the distance from z to z^2 (on the complex plane) is half the distance from 1 to z (on the complex plane), which in turn equals the distance from 1 to z^2. Compute b.

ANSWER TO PROBLEM 19

ANSWER TO PROBLEM 20

2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 21-22 **Name________________________________**

Time Limit: 10 Minutes **School_______________________________**

21. If a and k are constants, and if the system below has infinitely many solutions (x, y, z), compute the value of k.

$$\begin{cases} 4x + 3y = 15 \\ 2x + 5z = k \\ 3y - az = 4 \end{cases}$$

22. Compute the number of ordered pairs of integers (a, b) satisfying the equation

$$4a^2 + b = b^2 + 30.$$

ANSWER TO PROBLEM 21 **ANSWER TO PROBLEM 22**

2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 23-24 **Name**________________________________

Time Limit: 8 Minutes **School**_______________________________

23. If the points $(4, b)$, $(b, 2b - 8)$ and $(-4, 8)$ are collinear, compute all possible values of b.

24. Eleven points $A_1, A_2, \dots, A_{11}$ are arranged on a circle in that order, and connected by chords as follows: A_1 is connected to A_3, A_3 to A_5, … , A_9 to A_{11}, A_{11} to A_2, A_2 to A_4, … A_{10} to A_1. Compute the minimum possible value, in degrees, of $m\angle A_1 + m\angle A_2 + \dots + m\angle A_{11}$.

ANSWER TO PROBLEM 23

ANSWER TO PROBLEM 24

2009 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Part I Answers

1. (10,6)
2. $\frac{19}{4}$
3. $172\sqrt{11}$
4. 4
5. 150
6. $\frac{\pi}{20}$
7. $\frac{43}{19}$
8. 16°
9. 648
10. 12, 14, 18, or 20 (must have all four)
11. 102°
12. $\frac{\pi}{4}$
13. 3
14. 10

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Part II Answers

15. 336

16. $\frac{1033}{3094}$

17. 28

18.

1	2	1	1
2	8	6	0
8	7	2	0

19. $12\sqrt{3}$

20. $\frac{\sqrt{15}}{8}$

21. $\frac{11}{2}$

22. 8

23. 12 or 8 (must have both)

24. 1260°

Tryout Results 2008-09

198 students tried out. Top score was 18; cutoff for A/B was 11.

Item Analysis

Q	# right
1	44
2	116
3	22
4	138
5	64
6	9
7	78
8	60
9	12
10	30
11	38
12	27

Q	# right
13	80
14	101
15	40
16	4
17	95
18	64
19	25
20	2
21	92
22	8
23	153
24	37

Problems 4 and 12 were much easier than anticipated, because their answers were guessable.

Problems 16 and 20 were much more difficult than anticipated, because Problem 16 had so many cases, and problem 20 had a "red herring" solution if students tried to write equations starting with the form $a + bi$.