1996 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 1-2 Name________________________________

Time Limit 9 Minutes School________________________________

1. The number x > 0. The set of numbers 3, 36, and x has an arithmetic mean which is 13 greater than the geometric mean. Find the value of x.

2. In the figure shown, $\overline{AD} \cong \overline{BC}$

$\overline{MN} \perp \overline{DC}$

$\overline{MN} \perp \overline{AB}$

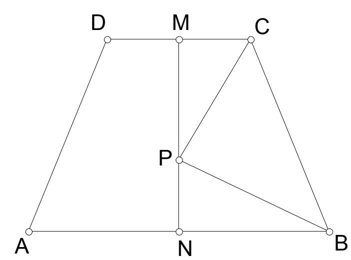
$\overline{PC} \perp \overline{PB}$

M is the midpoint of $\overline{\mathrm{DC}}$

N is the midpoint of $\overline{\mathrm{AB}}$

$CD = 8, AB = 10, MN = 12$

Find length PN



ANSWER TO PROBLEM 1

ANSWER TO PROBLEM 2

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Problems 3-4 **Name________________________________**

Time Limit 10 Minutes **School_______________________________**

3. Find the shortest altitude of a triangle whose sides have lengths 17, 25, and 28.

4. If $x = \sqrt[3]{20+14\sqrt{2}} + \sqrt[3]{20-14\sqrt{2}}$ **, find the value of** $x^3 - 6x$ **in simplest from.**

ANSWER TO PROBLEM 3

ANSWER TO PROBLEM 4

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Problems 5-6 Name________________________________

Time Limit 11 Minutes School________________________________

5. In triangle ACE, B is on AC and D is on CE, and BE meets AD at F. EB bisects angle AEC. If AB = 3, BC = 4, AE = 6, and F is the midpoint of BE, find CD.

6. Find all real numbers x, with $0 \le x < 2\pi$, such that sin x, sin 2x, sin 3x form an arithmetic sequence.

ANSWER TO PROBLEM 5

ANSWER TO PROBLEM 6

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Problems 7-8 **Name**________________________________

Time Limit 11 Minutes **School**_______________________________

7. A six-digit number consists of the same three digits repeated twice [such as 732732]. Find the least possible number of positive divisors such a six-digit number can have.

8. Acute triangle ABC is inscribed in a circle. Altitudes AM and CN are extended to meet the circle at P and Q, respectively. If PQ:AC = 7:2, find $\sin\angle ABC$.

ANSWER TO PROBLEM 7

ANSWER TO PROBLEM 8

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Problems 9-10 **Name**__________________________________

Time Limit 12 Minutes **School**__________________________________

9. The roots of $x^3 + 2kx^2 - kx + 10 = 0$ **form an arithmetic progression. All the roots are integers. Find the value of** k**.**

10. If $\sin^6 x + \cos^6 x = \frac{2}{3}$ **, find all possible values of** $\sin(2x)$

ANSWER TO PROBLEM 9

ANSWER TO PROBLEM 10

Problems 11-12 **Name**_______________________________

Time Limit 13 Minutes **School**______________________________

11. **Trapezoid ABCD is inscribed in a circle with diameter AB. Triangle ABC has area 150 and triangle ACD has area 120. Find length BC.**

12. **A fair coin is tossed 17 times consecutively. Find the probability that there will be at least 11 consecutive heads.**

ANSWER TO PROBLEM 11

ANSWER TO PROBLEM 12

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Problems 13-14 **Name**_______________________________

Time Limit 9 Minutes **School________________________________**

13. Find the ordered quadruple of positive integers (a,b,c,d)

such that $$\frac{423}{98} = a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d}}}$$

14 Find all positive intgers n $\le$ 20 such that the set {1,2,3,...,n} can be partitioned into two disjoint subsets so that the sum of the elements in one subset is twice the sum of the elements in the other subset.

ANSWER TO PROBLEM 13

ANSWER TO PROBLEM 14

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Problems 15-16 **Name__________________________________**

Time Limit 10 Minutes School________________________________

15. In the sequence 6, x, y, 16, the first three are in arithmetic progression and the last three are in geometric progression. Find all such ordered pairs (x,y).

16. Triangle ABC is located in the interior of triangle DEF. The sides of triangle ABC are parallel to and 2 units away from the sides of triangle DEF. If triangle DEF has sides 13, 14, and 15, find the area between the two triangles.

ANSWER TO PROBLEM 15

ANSWER TO PROBLEM 16

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Problems 17-18 Name_________________________________

Time Limit 10 Minutes School________________________________

17. Solve for x: $(2+\log_2 x)^3 + (-1+\log_2 x)^3 = (1+\log_2 x^2)^3$

18. Complete the cross-number puzzle at right in which each across answer is a four-digit positive integer and each down answer is a three-digit positive integer.
[Note: the grid at right is for scratch work only. The answer must appear in the answer space at the bottom of the page.]

1	2	3	4
5			
6			

Across

1. Last two digits are equal

5. A Fibonacci number with a digit that appears twice

6. Digits all distinct multiples of the same prime digit

Down

1. A perfect cube

2. A Fibonacci number

3. Digits strictly increasing in a perfect square

4. A perfect cube

ANSWER TO PROBLEM 17

ANSWER TO PROBLEM 18

1	2	3	4
5			
6			

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Problems 19-20 Name________________________________

Time Limit 12 Minutes School________________________________

19. Find the value of $\tan 20° + \tan 40° + \sqrt{3}\tan 20° \tan 40°$

20. In parallelogram ABCD, angle A is acute and AB = 5. Point E is on side AD with AE = 4 and BE = 3. A line through B, perpendicular to line CD, intersects line CD at point F. If BF = 5, find length EF.

ANSWER TO PROBLEM 19

ANSWER TO PROBLEM 20

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Problems 21-22 **Name**__________________________________

Time Limit 10 Minutes **School**__________________________________

21. **In triangle ABC, AC = 3, BC = 4, and AB = 5. A circle with center on side AC is tangent to sides AB and BC. Find the area of the circle.**

22. **The coordinates of the vertices of a triangle, in polar coordinates, are (12,30°), (4,90°), and (8,150°). Find the area of the triangle.**

ANSWER TO PROBLEM 21	**ANSWER TO PROBLEM 22**

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Problems 23-24 **Name**________________________________

Time Limit 10 Minutes **School**________________________________

23. Simplify completely: $\left(\log_4 27\right)\left(\log_9 625\right)\left(\log_5 16\right)$

24. In rectangle ABCD, AB = 6 and BC = 8. Equilateral triangles ADE and DCF are drawn on the exterior of ABCD. Find the area of triangle BEF.

ANSWER TO PROBLEM 23

ANSWER TO PROBLEM 24

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ANSWERS 1-14

1. 54

8. $\frac{1}{4}$

2. $2,10(\textit{any order})$

9. -3

3. 15

4. 40

5. $\frac{28}{5}$

6. $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

7. 16

10. $\frac{2}{3}, \frac{-2}{3}$

11. 10

12. $\frac{1}{512}$

13. (4,3,6,5)

14. 2,3,5,6,8,9,11,12,14,15,17,18,20

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ANSWERS 15-24

15. $(9,12),(1,-4)$

16. 63

17. $\frac{1}{4}, 2, \frac{\sqrt{2}}{2}$

18. $\begin{matrix} 3 & 6 & 2 & 2 \\ 4 & 1 & 8 & 1 \\ 3 & 0 & 9 & 6 \end{matrix}$

19. $\sqrt{3}$

20. $\sqrt{10}$

21. $\frac{16\pi}{9}$

22. $4\sqrt{3}$

23. 12

24. $36+25\sqrt{3}$